2019 ALABAMA COURSE OF STUDY: MATHEMATICS

Resources Utilized in the Development of the 2019 *Alabama Course of Study: Mathematics*

- The *2016 Revised Alabama Course of Study: Mathematics*
- The results from the Alabama Strategic Mathematics Planning Committee (2017)
- National Council of Teachers of Mathematics (NCTM) *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (2018)
- NCTM *Principles to Actions: Ensuring Mathematical Success For All* (2014)
- Documents from Other States
- Public and Professional Input

2019 Alabama Course of Study: Mathematics

- **Course of Study Committee**
- **Course of Study Task Force**

2019 ALABAMA COURSE OF STUDY: MATHEMATICS

CONCEPTUAL FRAMEWORK

Student Mathematics Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Content Standards

- Grades K – 8 are organized by Domain
- Grades 9 – 12 are organized by Essential Content

CONCEPTUAL FRAMEWORK

Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Position Statements

- Access and Equity in Mathematics Education
- Teaching and Learning Mathematics
- Mathematics Curriculum
- Mathematical Tools & Technology
- Assessment of Mathematics Learning
- Professional Mathematics Teachers

THE *2019 DRAFT ALABAMA COURSE OF STUDY: MATHEMATICS*

Minimum Required Content is Organized by Grade-Bands:

1. **Grades K – 2**
2. **Grades 3 – 5**
3. **Grades 6 – 8**
4. **Grades 9 – 12**

Grades K – 8

EACH GRADE HAS AN OVERVIEW
FOR EXAMPLE: KINDERGARTEN OVERVIEW

Domains	Counting and Cardinality (CC)	Operations and Algebraic Thinking (OA)	Number and Operations in Base Ten (NBT)	Measurement and Data (MD)	Geometry (G)
Clusters	• Know number names and the count sequence • Count to tell the number of objects • Compare numbers	• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from • Understand simple patterns	• Work with numbers 11 – 19 to gain foundations for place value	• Describe and compare measurable attributes • Classify objects and count the number of objects in each category	• Identify and describe shapes • Analyze, compare, create and compose shapes
Standards for Mathematical Practice	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.	5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

EACH GRADE LISTS THE CRITICAL AREAS OF FOCUS

GRADE 3 OVERVIEW

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. Please note that while every standard/topic in the grade level has not been included here, all standards should be included in instruction.

1. Through their learning in the ***Operations and Algebraic Thinking*** domain, students
   - develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size;
   - use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors; and
   - compare a variety of solution strategies, students learn the relationship between multiplication and division.

2. Through their learning in the ***Number and Operations - Fractions*** domain, students
   - develop an understanding of fractions, beginning with unit fractions;
   - view fractions in general as being built out of unit fractions, and use fractions along with visual fraction models such as area models, fraction strips, and number lines to represent parts of a whole;
   - understand that the size of a fractional part is relative to the size of the whole. Use fractions to represent numbers equal to, less than, and greater than one; and
   - solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Through their learning in the ***Measurement and Data*** domain, students
   - recognize area as an attribute of two-dimensional regions;
   - measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area; and
   - understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to determine the area of a rectangle.

4. Through their learning in the ***Geometry*** domain, students
   - extend knowledge of polygons to describe, analyze and compare properties of two-dimensional shapes;
   - recognize shapes that are/are not quadrilaterals by using informal language to classify shapes by sides and angles, and connect these with the names of the shapes; and
   - relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematics

Content Standard Strands

Domain →

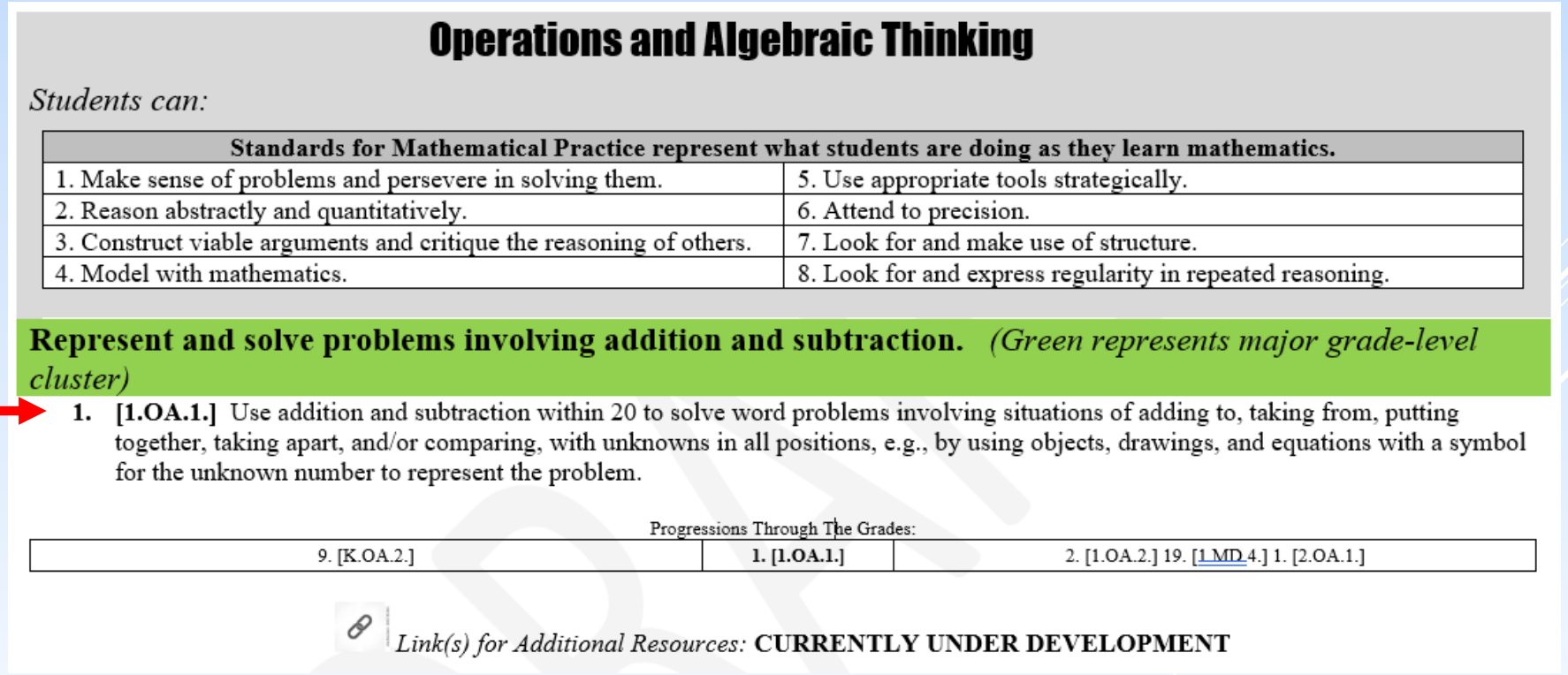
Standards for Mathematical Practice →

Cluster →

Alabama Standard Number →

Progression →

Additional Resources →



Operations and Algebraic Thinking

Students can:

Standards for Mathematical Practice represent what students are doing as they learn mathematics.	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

Represent and solve problems involving addition and subtraction. *(Green represents major grade-level cluster)*

1. **[1.OA.1.]** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and/or comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Progressions Through The Grades:

9. [K.OA.2.]	**1. [1.OA.1.]**	2. [1.OA.2.] 19. [1.MD.4.] 1. [2.OA.1.]

Link(s) for Additional Resources: **CURRENTLY UNDER DEVELOPMENT**

Support for School Districts

1. Appendices in Course of Study
2. ALEX Resources Planned
3. Professional Learning Partnerships
4. Model Programs

Mathematics Content Standard Strands

Domain

Operations and Algebraic Thinking

Students can:

Standards for Mathematical Practice represent what students are doing as they learn mathematics.	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

Mathematical Practice Standards

Cluster

Represent and solve problems involving multiplication and division. *(Green represents major grade-level cluster.)*

Alabama Standard Number

Content Standard Identifier

3. **[3.OA.3.]** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (Diagram will be added at a later date.)

Standard Progressions

Progression through the grades: CURRENTLY UNDER DEVELOPMENT

Additional Resources

Link(s) for Additional Resources: CURRENTLY UNDER DEVELOPMENT

Sample Progression - Fractions

Progression through the Grades (K-2)		
Kindergarten Embedded Standards	1st Grade Standard 22. [1.G.3.]	2nd Grade Standard 26. [2.G.2.]

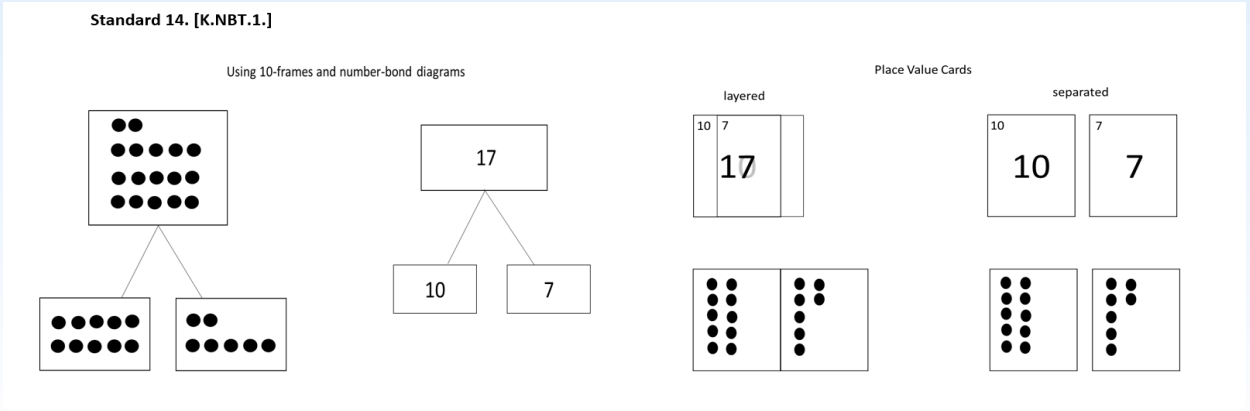
Sample Progression - Fractions

Progression through the Grades (3-5)		
3rd Grade Standard 13. [3.NF.1.]	4th Grade Standard 15. [4.NF.4.]	5th Grade Standard 17. [5.NF.7.]

Each grade-band has resources in the Appendix. Resources for Grades K – 2: Appendix C

Table 1: Mathematics Practice Standards

Practice	Explanation and Example
1. Make sense of problems and persevere in solving them.	Mathematically proficient students in Kindergarten begin to develop effective dispositions toward problem solving. In rich settings in which informal and formal possibilities for solving problems are numerous, young children develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). Using both verbal and nonverbal means, kindergarten students begin to explain to themselves and others the meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, kindergarten students begin to reason as they become more conscious of what they know and how they solve problems.
2. Reason abstractly and quantitatively.	Mathematically proficient students in Kindergarten begin to use numerals to represent specific amount (quantity). For example, a student may write the numeral "11" to represent an amount of objects counted, select the correct number card "17" to follow "16" on the calendar, or build a pile of counters depending on the number drawn. In addition, kindergarten students begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation (Mary has 3 bears. Juanita gave her 1 more bear. How many bears does Mary have altogether?), or a separating situation (Mary had 5 bears. She gave some to Juanita. Now she has 3 bears. How many bears did Mary give Juanita?). Using the language developed through numerous joining and separating scenarios, kindergarten students begin to understand how symbols (+, -, =) are used to represent quantitative ideas in a written format.

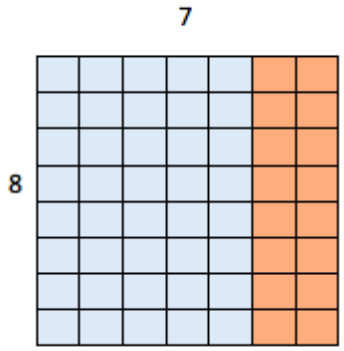


Each grade-band has resources in the Appendix.

Example of Appendix D:

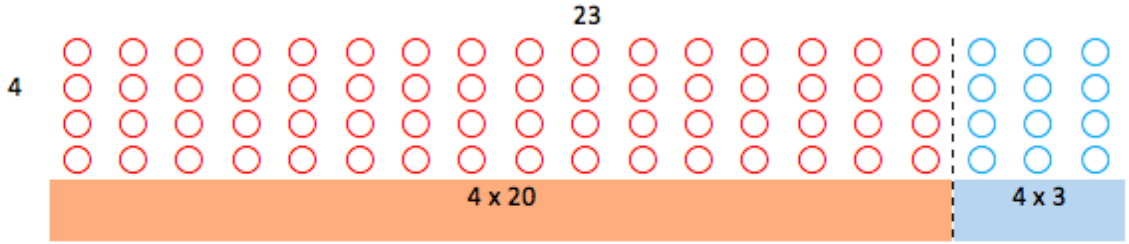
Resources for Grades 3 – 5:

Area Model to illustrate Distributive property and decomposing one factor to show the product of 8 x 7, a basic fact.



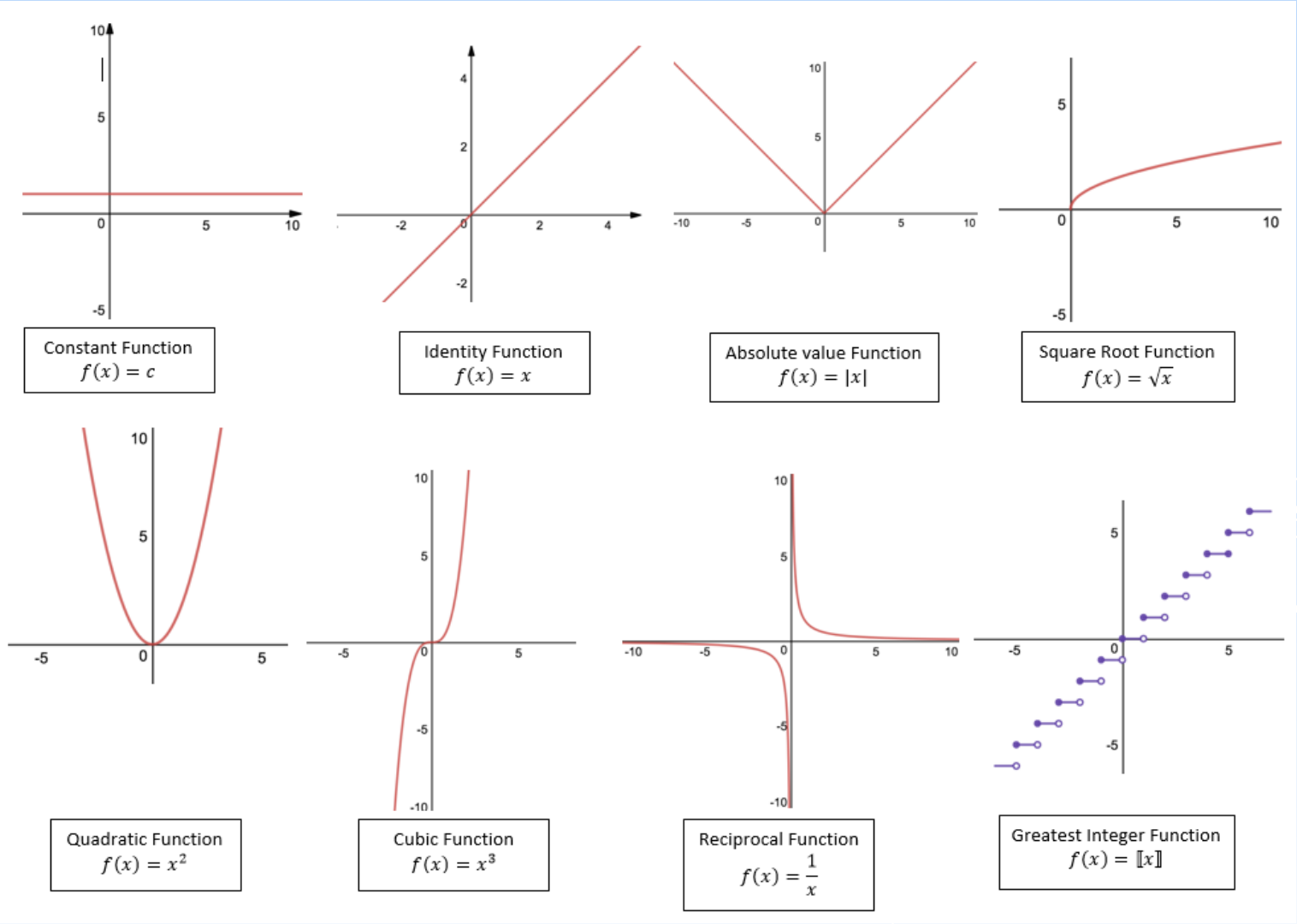
8 x 7 = (8 x 5) + (8 x 2)

Distributive Property Using an Array Model

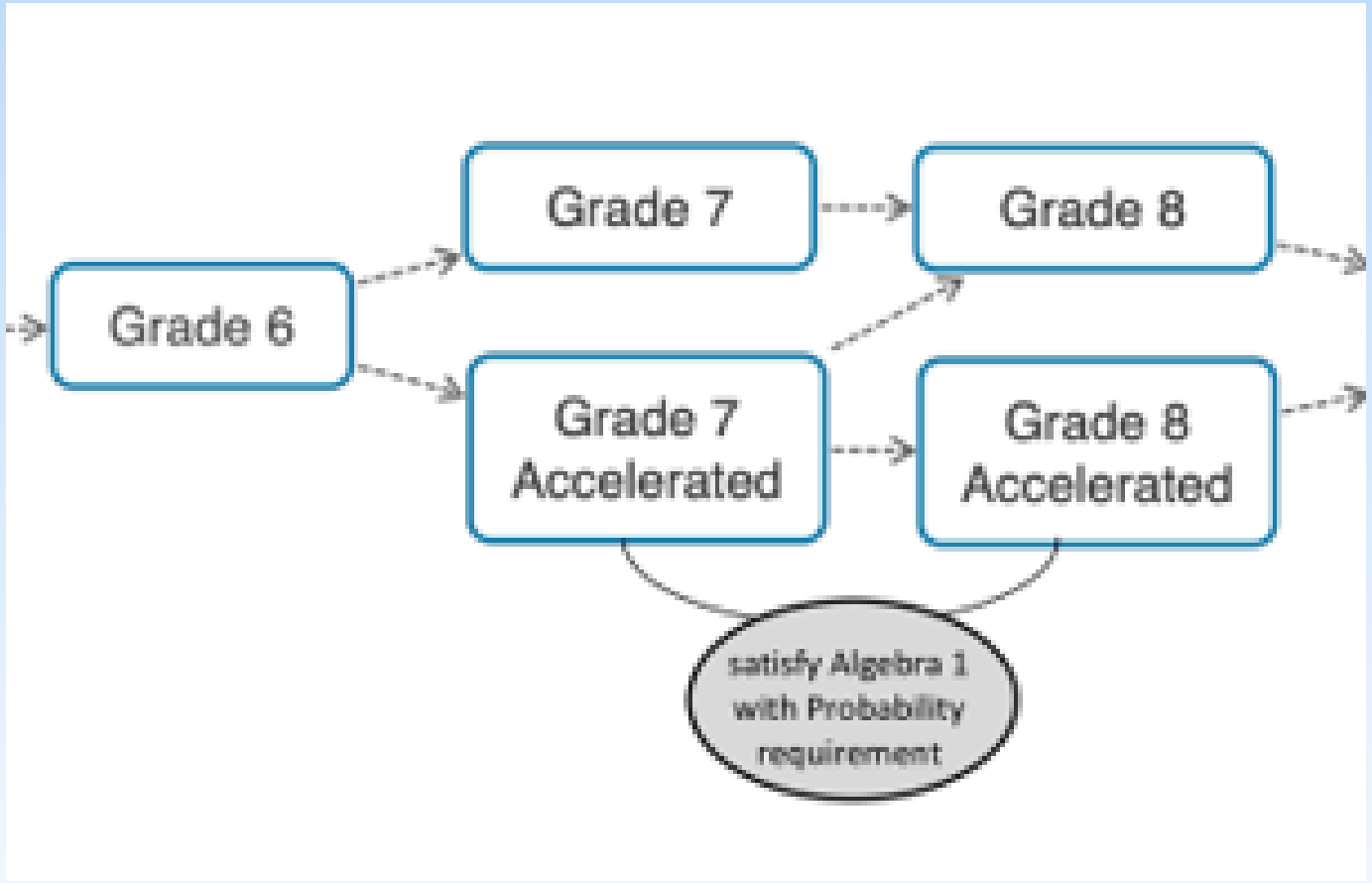


4 x 23 = (4 x 20) + (4 x 3)

Appendix E: Resources for Grades 6 – 8 Example: Table 6: Functions



Grades 6-8 Overview



Overview for Grades 7-8 Accelerated

- Accelerated courses have been carefully aligned and designed for middle school students who show particular motivation and interest in mathematics.

- Students meet all the standards of Grade 7, Grade 8, and Algebra I with Probability within the Grade 7 Accelerated and Grade 8 Accelerated courses, thus merging all the standards from three years of mathematics into two years.

- The 2019 Course of Study: Mathematics includes standards for Grade 7 Accelerated and Grade 8 Accelerated. Previously, school districts created accelerated classes with little guidance from the Alabama Department of Education. This practice led to varied acceleration models.

Overview for Grades 7-8 Accelerated, continued

- Students who successfully complete this pathway will be prepared to enter Geometry with Statistics in Grade 9, and then accelerate directly into Algebra II with Inferential Statistics in Grade 10, thus providing them with an opportunity to take additional specialized mathematics coursework, such as AP Calculus or AP Statistics.

- Grade 7 Accelerated includes a function content domain. The algebra content focus is on linear relationships.

- The algebra focus is on quadratic relationships in Grade 8 Accelerated.

- Grade 7 Accelerated is a prerequisite for Grade 8 Accelerated.

Domain

7th Accelerated

Functions

Students can:

Alabama Standard Number

15. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.) **[8.F.1.]**

16. Define, evaluate and compare functions.

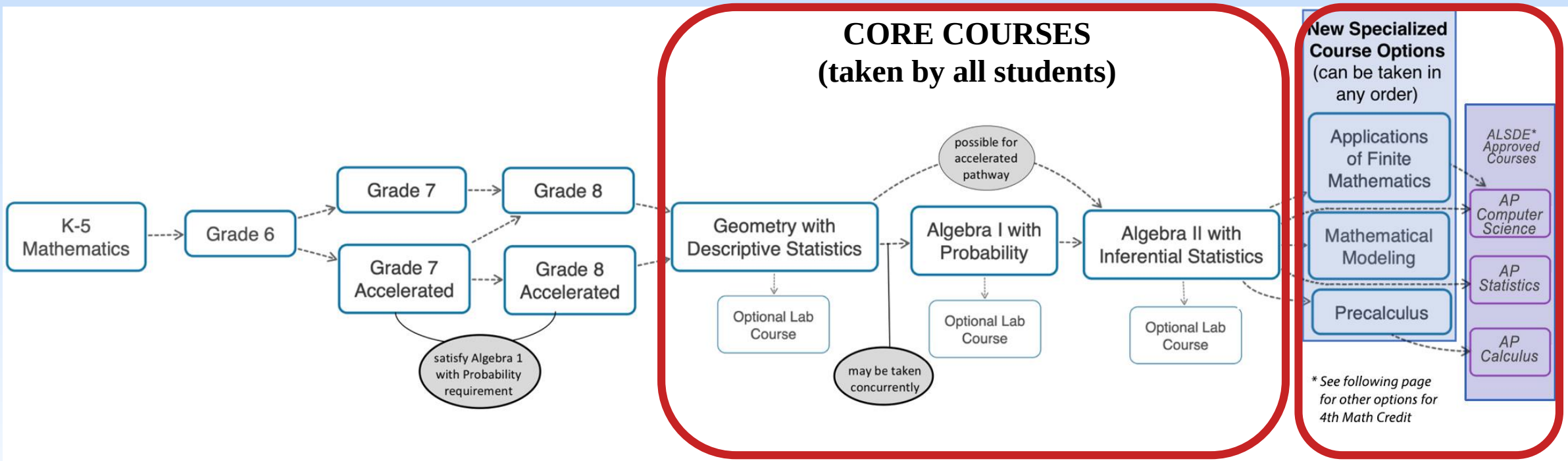
a. Interpret relationships as either functions or not functions.
b. Interpret relationships as either liner or nonlinear.
c. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **[8.F.3] [A.REI.10.]**

Content Standards

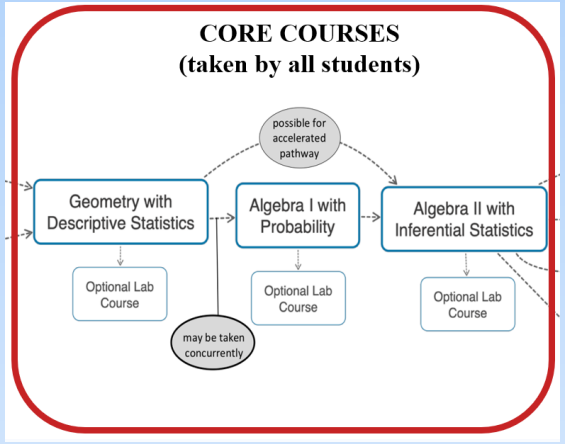
Content Standard Identifier

Grades 9 – 12 OVERVIEW

Pathways to Student Success: including the postsecondary study of mathematics, careers, and lifelong use and enjoyment of mathematics.



Grade 9	Grade 10	Grade 11	Grade 12
Geometry with Descriptive Statistics	**Algebra I with Probability**	**Algebra II with Inferential Statistics**	*Specialized Course*
Geometry with Descriptive Statistics AND Algebra I with Probability	**Algebra II with Inferential Statistics**	*Specialized Course*	*Specialized Course*
Geometry with Descriptive Statistics	**Algebra II with Inferential Statistics** *(with successful completion of accelerated middle grades)*	*Specialized Course*	*Specialized Course*



Content Domain →

Focus Area →

Essential Concept →

Standard →

Essential Concepts in Algebra and Functions

Focus 1: Algebra

The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.

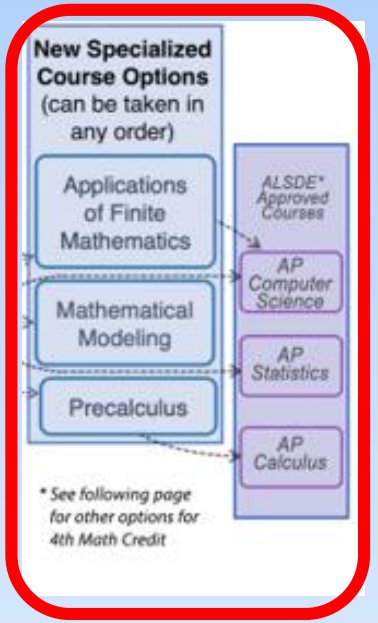
1. Find the coordinates of the vertices of a polygon formed by a set of lines, given their equations, by setting their linear functions rules ($mx + b$) equal and solving, or by using their graphs. Example: Find the coordinates the vertices of the triangle formed by three lines given their equations. **[A.REI.6. edited]**

Content Standard Identifier

ESSENTIAL CONCEPTS

- The Essential Concepts are defined by the National Council of Teachers of Mathematics (NCTM, 2018) in *Catalyzing Change in High School Mathematics.*
- The Essential Concepts include “the concepts and skills that all students need to build the mathematical foundation necessary for the continued study of mathematics and to achieve the multiple purposes for learning mathematics.” (p. 37)
- **Page with Essential Concepts..**

Grade 9	Grade 10	Grade 11	Grade 12
Geometry with Descriptive Statistics	Algebra I with Probability	Algebra II with Inferential Statistics	***Specialized Course***
Geometry with Descriptive Statistics AND Algebra I with Probability	Algebra II with Inferential Statistics	***Specialized Course***	***Specialized Course***
		Precalculus	**AP Calculus**
Geometry with Descriptive Statistics	Algebra II with Inferential Statistics *(with successful completion of accelerated middle grades)*	***Specialized Course***	***Specialized Course***
		Precalculus	**AP Calculus**



Conceptual Category → **FUNCTIONS**

Domain → **Interpreting Functions**

Cluster → **Interpret functions that arise in applications in terms of the context.**

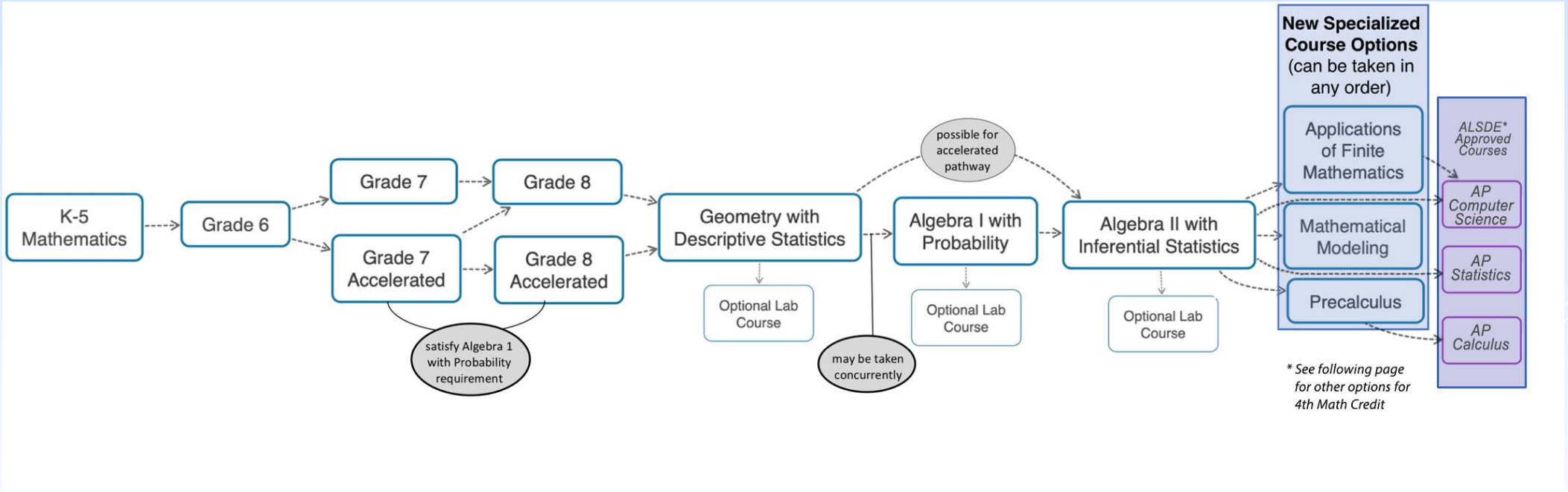
Standard →

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither. Extend analysis to include all trigonometric, rational, and general piecewise-defined functions. **[F.IF.4]**

↑ Content Standard Identifier

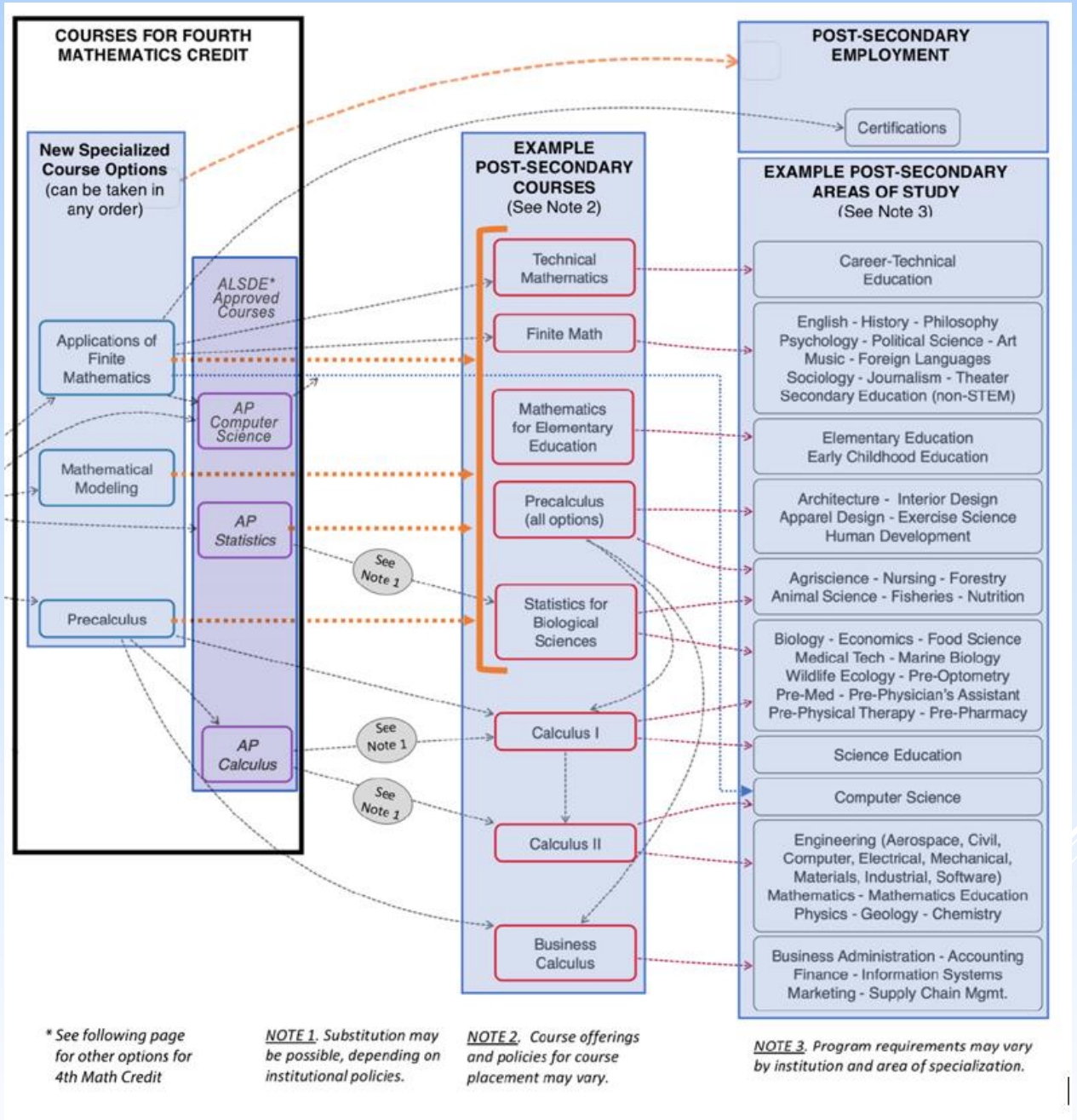
APPENDIX B


POSITION STATEMENTS
MATHEMATICS TEACHING PRACTICES
STUDENT MATHEMATICS PRACTICES
K-8 DOMAINS OF STUDY
9-12 ESSENTIAL CONTENT


K-5 Mathematics
Grade 6
Grade 7
Grade 8
Grade 7 Accelerated
Grade 8 Accelerated
satisfy Algebra 1 with Probability requirement
Geometry with Descriptive Statistics
possible for accelerated pathway
Optional Lab Course
Algebra I with Probability
may be taken concurrently
Optional Lab Course
Algebra II with Inferential Statistics
Optional Lab Course
New Specialized Course Options (can be taken in any order)
Applications of Finite Mathematics
Mathematical Modeling
Precalculus
* See following page for other options for 4th Math Credit
ALSDE* Approved Courses
AP Computer Science
AP Statistics
AP Calculus

APPENDIX B


POSITION STATEMENTS
MATHEMATICS TEACHING PRACTICES
STUDENT MATHEMATICS PRACTICES
K-8 DOMAINS OF STUDY
9-12 ESSENTIAL CONTENT


COURSES FOR FOURTH MATHEMATICS CREDIT
New Specialized Course Options (can be taken in any order)
ALSDE* Approved Courses
Applications of Finite Mathematics
AP Computer Science
Mathematical Modeling
AP Statistics
Precalculus
AP Calculus
See Note 1
See Note 1
See Note 1
EXAMPLE POST-SECONDARY COURSES (See Note 2)
Technical Mathematics
Finite Math
Mathematics for Elementary Education
Precalculus (all options)
Statistics for Biological Sciences
Calculus I
Calculus II
Business Calculus
POST-SECONDARY EMPLOYMENT
Certifications
EXAMPLE POST-SECONDARY AREAS OF STUDY (See Note 3)
Career-Technical Education
English - History - Philosophy
Psychology - Political Science - Art
Music - Foreign Languages
Sociology - Journalism - Theater
Secondary Education (non-STEM)
Elementary Education
Early Childhood Education
Architecture - Interior Design
Apparel Design - Exercise Science
Human Development
Agriscience - Nursing - Forestry
Animal Science - Fisheries - Nutrition
Biology - Economics - Food Science
Medical Tech - Marine Biology
Wildlife Ecology - Pre-Optometry
Pre-Med - Pre-Physician's Assistant
Pre-Physical Therapy - Pre-Pharmacy
Science Education
Computer Science
Engineering (Aerospace, Civil, Computer, Electrical, Mechanical, Materials, Industrial, Software)
Mathematics - Mathematics Education
Physics - Geology - Chemistry
Business Administration - Accounting
Finance - Information Systems
Marketing - Supply Chain Mgmt.
* See following page for other options for 4th Math Credit
NOTE 1. Substitution may be possible, depending on institutional policies.
NOTE 2. Course offerings and policies for course placement may vary.
NOTE 3. Program requirements may vary by institution and area of specialization.

2019 ALABAMA COURSE OF STUDY: MATHEMATICS

Thank You!